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A Buyer-Vendor EOQ model with time varying holing cost involving lead time as a decision variable under an Integrated Supply Chain system

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Abstract— In this paper, a supplier retailer inventory model is developed. Here, we describe an EOQ model with changeable lead-time and time dependent holding cost under an Integrated Supply Chain System. This situation is very common in the market , once an enterprise has some key technology or product that others have not, as a supplier, it can decide the prices and lead time of the technology or product to the buyers or retailers according to its need . Then the retailer determines his optimal order strategy , i.e., decides on the quantity of products to order from the supplier . Under this circumstance , the problem that lead time, as a controllable variable of the supplier , and how it affects the cost to the supplier, retailer and whole supply chain is very important to the supplier and retailer because double-win benefits is a base of existence for the supply chain . In this discussion , with a fixed market and lead time as a controllable variable , Retailer's Optimal Cost , Supplier's Optimal Cost , the optimal lead time of the supplier and order cycle time of the retailer, respectively, are investigated and approximate solutions for them are derived. Numerical examples are presented to compare results between the different demand rates and solved by using LINGO software. The Sensitivity analysis is carried out to analyze the effect of critical parameters on the optimal solutions.

Index Terms— EOQ , Time-dependent holding cost , Lead time , Integrated System, Supply chain, LINGO Software.

1 Introduction

In the last few decades, extensive researches have been performed in the area of supply chain coordination. A Supply Chain is a sequence of processes and flows that take place within and between different stages and combine to fill a customer need for a product .Traditionally, marketing, distribution, planning, manufacturing, and the purchasing organizations along the supply chain operated independently. Supply chains exist in both service and manufacturing organizations, although the complexity of the chain may vary greatly from industry to industry and firm to firm . Many manufacturing operations are designed to maximize throughput and lower costs with little consideration for the impact on inventory levels and distribution capabilities . Supply Chain Management is the combination of art and science that goes into improving the way your company finds the raw components it needs to make a product or service and deliver it to customers .

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Supply Chain Management is a business practice that aims to improve the way a business sources its raw materials , and delivers it to end users . For any product or service offered by any business , there are usually a number of different business entities involved the various stages of the Supply-Chain, including manufacturers, whole sellers, distributors and retailers, the last group in a Supply-Chain is consumers .

The Economic Order Quantity (EOQ) model, where cumulative holding cost is a convex function of time, is in contrast with the classic EOQ model where holding cost is a linear function of time. More specifically, the cumulative holding cost for *one* unit that has been stored during *t* units of time is $H(t) = C_1 t^n$, where C_1 and n (≥ 1) are constants. If n=1, then the problem reduces to the classic EOQ model with C_1 being the cost to hold one unit for one time period. Mark Ferguson et al. [6] studied a note: An Application of the EOQ Model with Nonlinear Holding Cost to Inventory Management of Perishables . Vinod Kumar Mishra [11] developed a Inventory model for time dependent holding cost and deterioration with salvage value and shortages. G.C. Mahata and A. Goswami [2] has used fuzzy concepts to develop a fuzzy EOQ model with stock-dependent demand rate and non-linear holding cost by taking rate of deterioration to be a triangular fuzzy number. In recent years, EOQ research has many new directions, such as economic order quantity with random supplier capacity, quantity discounts for the vendor's benefit and the buyer-vendor coordination of inventory, ordering, pricing, etc. The aforementioned studies all assumed that the supplier's lead time is zero, or constant, that is to say, no stockouts are permitted in the model. S. Viswanathan [10] explained Optimal strategy for the integrated vendor-buyer inventory model. S.K. Goyal , and Y.P. Gupta. [9] reviewed Integrated inventory models: the buyer-vendor coordination. P. Piplani and S. Viswannathan [8] Coordinated supply chain inventories through common replenishment epochs. The traditional EOQ (economic order quantity) model focuses on the buyer's view and makes several assumptions, e.g., fixed market demand rate, no stockouts , unlimited supplier capacity and zero lead time, which are far from the actual requirements. In practice , such assumptions are rather difficult to realize . For instance, when the vendor has difficulty in supplying , there will be a deferred delivery problem and, as a result , the retailer can't replenish his inventory instantaneously. Another case is , when the supplier holds a monopolistic status , he will not only control products' prices but also their lead time in order to obtain the highest profits and the lowest cost . some researchers take into account the factor stockout in their research on the EOQ model and the lead time as a decision variable of the buyer . Liao and Shyu [1] proposed a model that can be used to determine the length of lead time that minimizes the expected total relevant cost for a continuous review policy where the order quantity and reorder level are known. The lead time is the only decision variable in many researchers' model . Hariga and Ben-Dayu [7] construct a continuous review inventory model where the reorder point, the ordering quantity and the lead time are decision variables. Models with full and partial information about the lead-time demand distributions are developed . J.M. Hsiao and C. Lin [3] considered a buyer-vendor EOQ model with changeable lead-time in supply chain". Yung-Fu Huang et al. [12] explained an easy approach to derive a buyer-vendor EOQ model with changeable lead-time in supply chain . Ben-Dayu and Raouf [5] constructed a model that considers both lead time and order quantity as decision variables . Later, Ouyang et al. [4] extended Ben-Dayu and Raouf's model considering shortages where the total amount of stockouts is considered as a mixture of backorders and lost sales. They assumed a given service level and therefore the reorder point is fixed.

In this article , we consider an EOQ model involving lead time as a decision variable with time varying holding cost on an Integrated System in the supply chain .That is , the distribution channel system contains one supplier and a single retailer and the supplier in the channel holds monopolistic status , in which he not only owns cost information about the retailer , but also has the decision-making right of the lead time . For example, the Ford Company sells automobiles to its dealerships, which in turn retail the automobiles to their potential customers. In this case, the automobile manufacturer represents the supplier and the dealerships represent the buyers. For those cases, the Ford Company is in a monopolistic position with respect to the dealerships. The supplier can decide its products' prices and lead time according to the situation, and the buyer then decides on the quantity of the products to order from the supplier, based on the supplier's products' prices and lead time. This situation is very common in the market . In this discussion , with a fixed market and lead time as a controllable variable , the optimal lead time of the supplier and order cycle time of the retailer, respectively, are investigated . The problem is then solved by using LINGO software.

2 Assumptions and Notations

We present here a continuous review, deterministic inventory model under the following assumptions and notations:

2.1 Notations and the basic model of the Retailer

The notations are defined as follows:

- Q : The retailer's initial inventory level (quantity)
- $H(t)$: The retailer's holding cost per product
- C_2 : The retailer's stockout cost per product
- C_3 : The retailer's setup cost for each order
- $D(t)$: Market's demand rate for some product
- $f(t)$: Total market's demand rate for some product from the beginning to time t, $f(t) = \int_0^t D(t)dt$.
- T : The retailer's order cycle time, a decision variable

Here the holding cost per unit increases with the time t that the product has been in stock according to $H(t) = C_1 t^n$, where C_1 and $n \geq 1$ are constants. Suppose the retailer's order cycle time is t with no stockouts , then the average cost of the retailer in t is

$$TC_r(t) = \frac{C_1}{t}\int_0^t t^n f(t)dt + \frac{C_3}{t}, \quad (1)$$

where $\frac{C_1}{t}\int_0^t t^n f(t)dt$ is the retailer's average holding cost in order cycle time *t*,
$\frac{C_3}{t}$ is the retailer's average setup cost in order cycle time *t*.

2.2 Notations and the basic model of the Supplier

The notations are defined as follows:

A : The supplier's setup cost for each order
h(*t*) : The supplier's holding cost per product
L : The supplier's lead time, a decision variable, $L < T$

Here the holding cost is a linear function of time *t* as $h(t) = h_1 + h_2 t$ where $h_1, h_2 > 0$. Then the average cost of the supplier for dealing with his order is

$$TC_s(t) = \frac{1}{t}\int_0^t (h_1 + h_2 t) f(t)dt + \frac{A}{t}, \quad (2)$$

where $\frac{1}{t}\int_0^t (h_1 + h_2 t) f(t)dt$ is the supplier's average holding cost in order cycle time *t*,
$\frac{A}{t}$ is the supplier's average setup cost in order cycle time *t*.

3 Cost models of the retailer and the supplier

In the distribution channel system with one seller and one buyer, in the buyer-vendor relationship , the supplier holds monopolistic status (leader) and the retailer is the follower . The leader, who has the ability to impose its strategies on the other player , declares its strategies first and imposes them on the follower . The follower then reacts to the leader's actions and decides on its strategies . when retailer's inventory is zero , he issues orders to the supplier immediately . However, the supplier always delivers the goods to him after a span of *L* in order to get the biggest profits or for other reasons , that causes the retailer to be out of stock . As a result , customers have to buy the goods elsewhere , which is called loss sale . Additionally , the supplier possesses the information about the retailer's ordering and inventory cost , the supplier first announces whole sale price and lead time , the retailer then determines his optimal order strategy , i.e., decides on the quantity of products to order from the supplier .

(i) Retailer's average total cost in cycle *T* :

Assume that the retailer's initial inventory level is Q, which can meet the demand for time ($T - L$), $f(t) = Q - \int_0^t D(t)dt$, during which the average inventory is $\frac{1}{T-L}\int_0^{T-L} f(t)dt$. The inventory in time *L* is zero and its average stockout is $S = \frac{1}{L}\int_{T-L}^{T} f(t)dt$. For *Q* is only enough for demand over time ($T - L$), then $Q = \int_0^{T-L} D(t)dt$, as shown in Fig. 1.

Thus, the retailer's average total cost in cycle *T* is

$$C_r(T,L) = \frac{1}{T}\left(C_1\int_0^{T-L} t_n f(t)dt - C_2\int_{T-L}^{T} f(t)dt + C_3\right) \quad (3)$$

(ii) Supplier's average total cost in cycle *T* :

The supplier's setup cost of orders is *A* , from beginning to time ($T - L$) , the retailer has some inventory and suppliers have to hold a quantity of products, which equals the market's demand for the product, i.e., that of the retailer's holding (see Fig. 1), so the supplier's holding cost is $\int_0^{T-L}(h_1 + h_2 t) f(t)dt$.

Thus, the supplier's average total cost in cycle *T* is

$$C_s(T,L) = \frac{1}{T}\left(\int_0^{T-L}(h_1 + h_2 t) f(t)dt + A\right) \quad (4)$$

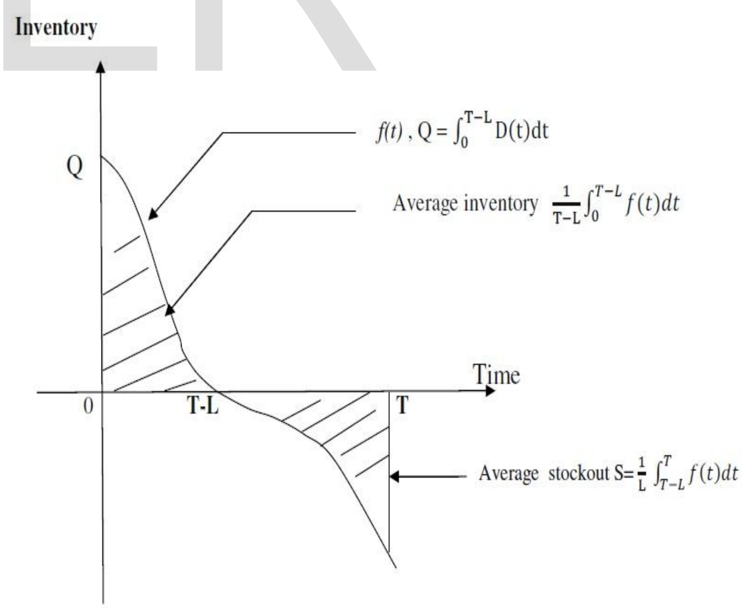

Fig. 1. The structure of the model

Combining Equations (3) and (4) ,we get the total average cost as follows

$$TC(T,L) = C_r(T,L) + C_s(T,L) \quad (5)$$

4 Evaluating Average Total Cost for different demand rates

(i) $D(t) = ae^{bt}$, where $0 < b < 1$, $t > 0$, $a > 0$, the market's demand rate for some product is exponentially increasing in nature.

Here $f(t) = Q - \int_0^t D(t)dt = Q + \frac{a}{b}(1 - e^{bt})$.

From $f(T-L) = 0$, i.e. , Q is only enough for demand over time $(T-L)$, then we get

$Q = \frac{a(e^{b(T-L)}-1)}{b}$.

Thus , the retailer's average total cost in cycle T from (3) is

$$C_r(T,L) = \frac{1}{T}[C_1 \int_0^{T-L} t^n \left\{Q + \frac{a}{b}(1-e^{bt})\right\}dt - C_2 \int_{T-L}^{T} \left\{Q + \frac{a}{b}(1-e^{bt})\right\}dt + C_3]$$

$$= \frac{1}{T}\left[\frac{aC_1(T-L)^{n-2}}{n+1}\left\{\frac{1}{n+2} + \frac{b(T-L)}{2}\right\} + \frac{aC_2L^2}{2}\left\{1 + b(T - \frac{2L}{3})\right\} + C_3\right]. \tag{6}$$

and the supplier's average total cost in cycle T from (4) is

$$C_s(T,L) = \frac{1}{T}\left(\int_0^{T-L}(h_1 + h_2 t)\left\{Q + \frac{a}{b}(1 - e^{bt})\right\}dt + A \right)$$

$$= \frac{1}{T}\left[h_1\left\{\frac{a(T-L)^2}{2} + \frac{ab}{3}(T-L)^3\right\} + \frac{ah_2(T-L)^3}{24}\{4 + 3b(T-L)\}\right]. \tag{7}$$

Therefore, the total average cost in cycle T from (5) is

$$TC(T,L) = \frac{1}{T}\left[\frac{aC_1(T-L)^{n-2}}{n+1}\left\{\frac{1}{n+2} + \frac{b(T-L)}{2}\right\} + \frac{aC_2L^2}{2}\left\{1 + b(T - \frac{2L}{3})\right\} + C_3\right] + \frac{1}{T}\left[h_1\left\{\frac{a(T-L)^2}{2} + \frac{ab}{3}(T-L)^3\right\} + \frac{ah_2(T-L)^3}{24}\{4 + 3b(T-L)\}\right]. \tag{8}$$

(ii) $D(t) = \alpha\beta\, t^{\beta-1}$; $0 < \alpha < 1$, $\beta > 0$, $t>0$, the market's demand rate for some product follows a two-parameter Weibull distribution.

Here $f(t) = Q - \int_0^t D(t)dt = Q - \alpha t^{\beta}$, from $f(T-L) = 0$, we get $Q = \alpha(T-L)^{\beta}$.

Thus, the retailer's average total cost in cycle T from (4) is

$$C_r(T,L) = \frac{1}{T}[C_1 \int_0^{T-L} t^n (Q - \alpha t^{\beta})dt - C_2 \int_{T-L}^{T}(Q - \alpha t^{\beta})dt + C_3]$$

$$= \frac{1}{T}\left[\frac{C_1\alpha\beta(T-L)^{n+\beta+1}}{(n+1)(n+\beta+1)} + \frac{C_2\{\alpha T^{\beta+1} - \alpha(T+L\beta)(T-L)^{\beta}\}}{(\beta+1)} + C_3\right]. \tag{9}$$

and the supplier's average total cost in cycle T from (5) is

$$C_s(T,L) = \frac{1}{T}\left[\int_0^{T-L}(h_1 + h_2 t)(Q - \alpha t^{\beta})dt + A\right]$$

$$= \frac{1}{T}\left[\frac{h_1\alpha\beta(T-L)^{\beta+1}}{\beta+1} + \frac{h_2\alpha\beta(T-L)^{\beta+2}}{2(\beta+2)} + A\right]. \tag{10}$$

Therefore, the total average cost in cycle T from (5) is

$$TC(T,L) = \frac{1}{T}\left[\frac{C_1\alpha\beta(T-L)^{n+\beta+1}}{(n+1)(n+\beta+1)} + \frac{C_2\{\alpha T^{\beta+1} - \alpha(T+L\beta)(T-L)^{\beta}\}}{(\beta+1)} + C_3\right] + \frac{1}{T}\left[\frac{h_1\alpha\beta(T-L)^{\beta+1}}{\beta+1} + \frac{h_2\alpha\beta(T-L)^{\beta+2}}{2(\beta+2)} + A\right]. \tag{11}$$

5 Optimal Solutions L and T of the models

The objective in the following is to find the solutions for the optimal values of T and L (say T^* and L^*) that minimize the total average cost $TC(T,L)$.

The necessary condition for minimization of $TC(T,L)$ are

$$\frac{\partial TC(T,L)}{\partial T} = 0 \quad and \quad \frac{\partial TC(T,L)}{\partial L} = 0 \tag{12}$$

The sufficient condition for minimization of $TC(T,L)$ requires that it must be a convex function for $T>0$, $L>0$.

Now, the function $TC(T,L)$ will be convex if

$$\begin{vmatrix} \frac{\partial^2 TC(T,L)}{\partial T^2} & \frac{\partial^2 TC(T,L)}{\partial T \partial L} \\ \frac{\partial^2 TC(T,L)}{\partial L \partial T} & \frac{\partial^2 TC(T,L)}{\partial L^2} \end{vmatrix} > 0. \tag{13}$$

Equations(12) can be solved simultaneously by some computer oriented numerical technique to obtain retailer's optimal order cycle time T^* and supplier's optimal lead time L^* . For this, we have to prescribe the values of the parameters $C_1, C_2, C_3, A, h_1, h_2, \alpha, \beta, n$. As an illustration, we take up a numerical example.

6. Numerical Example

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(i) To illustrate the model for **exponential demand** ,the following example is considered.
Let $C_1 = 2, C_2 = 8, C_3 = 100, A = 200, h_1 = 0.2, h_2 = 0.1, a = 40, b = 0.5, n = 2$ in appropriate units.
Equations (8) and (12) are now solved simultaneously for the above parameter values using a gradient based non-linear optimization technique (LINGO) and get the results shown in Table 1. It is verified that all the solutions in Table 1 for different values of n, satisfy the convexity condition for *TC(T,L)*.

Table 1 Optimal solutions for various values of 'n'.

n	Retailer's optimal order cycle time T^*	Supplier's optimal lead time L^*	Retailer's Optimal Cost C_r^*	Supplier's Optimal Cost C_s^*	Total Optimal Cost TC^*
2	1.7954	0.4069	114.6600	119.1927	233.8527
3	1.7495	0.4088	110.2320	121.6438	231.8758
4	1.7135	0.4127	107.9547	123.6550	231.6097
5	1.6852	0.4170	106.6677	125.2994	231.9671
6	1.6627	0.4210	105.8972	126.6563	232.5536
7	1.6444	0.4247	105.4195	127.7898	233.2093
8	1.6292	0.4280	105.1180	128.7482	233.8661

(ii) To illustrate the model for **Weibull distribution demand rate** ,the following example is considered.
Let $C_1 = 2, C_2 = 8, C_3 = 100, A = 200, h_1 = 0.2, h_2 = 0.1, \alpha = 0.6, \beta = 20, n = 2$ in appropriate units.
Equations (9) and (12) are now solved simultaneously for the above parameter values using a gradient based non-linear optimization technique (LINGO) and get the results shown in Table 2. It is verified that all the solutions in Table.2 for different values of n, satisfy the convexity condition for *TC(T,L)*.

Table 2 Optimal solutions for various values of 'n'.

n	Retailer's optimal order cycle time T^*	Supplier's optimal lead time L^*	Retailer's Optimal Cost C_r^*	Supplier's Optimal Cost C_s^*	Total Optimal Cost TC^*
2	1.2257	0.1409	92.9812	163.8175	256.7987
3	1.2274	0.1268	92.5012	163.8281	256.3292
4	1.2285	0.1193	92.1502	163.8409	255.9911
5	1.2291	0.1152	91.8991	163.8516	255.7508
6	1.2295	0.1130	91.7219	163.8593	255.5812
7	1.2297	0.1119	91.5990	163.8642	255.4632
8	1.2297	0.1116	91.5618	163.8669	255.3831

7 Sensitivity Analysis

The sensitivity analysis is performed by changing the value of each of the parameters $C_1, C_2, C_3, A, h_1, h_2$ respectively by -70%, -50% , 50% and 70% , taking one parameter at each time and keeping the remaining parameters unchanged. We now study sensitivity of the optimal solution to changes in the values of the different parameters associated with the model. The results are shown in Table 3.

Table 3 The optimal solutions with different values of the parameters

Parameter	Changed parameter values	$D(t) = ae^{bt}$				
		T^*	L^*	C_r^*	C_s^*	TC^*
C_1	0.6 1	2.1065	0.3120	89.3957	107.7786	197.1743
	3	1.9641	0.3495	99.6348	112.2870	211.9219
	3.4	1.7100	0.4434	123.9962	123.4674	247.4635
		1.6855	0.4550	126.9501	124.8006	251.7507
C_2	2.4	2.3298	1.0540	101.6825	90.7064	192.3890
	4	2.0509	0.7188	108.6209	103.6686	212.2895
	12	1.6968	0.2851	116.7221	126.4718	243.1939
	13.6	1.6722	0.2547	117.1955	128.4158	245.6113
C_3	30	1.6621	0.3464	65.6984	127.6878	193.3562
	50	1.7026	0.3644	80.3156	124.9579	205.2735
	150	1.8783	0.4464	146.5377	114.5299	261.0676
	170	1.9093	0.4615	158.7328	112.8951	271.6279
A	60	1.4980	0.2775	129.7324	177.6080	149.1370
	100	1.5966	0.3181	102.3079	46.8291	174.9769
	300	1.9538	0.4835	125.3344	161.8241	287.1585
	340	2.0098	0.5117	129.7324	177.6080	307.3404
h_1	0.06	1.8035	0.3978	115.0085	114.3925	229.4010
	0.1	1.8012	0.4004	114.8990	115.7849	230.6839
	0.3	1.7895	0.4134	114.4698	122.4973	236.9670
	0.34	1.7871	0.4159	114.4070	123.7907	238.1977
h_2	0.03	1.7997	0.4044	114.8588	117.9285	232.7873
	0.05	1.7984	0.4051	114.8004	118.2929	233.0933
	0.15	1.7923	0.4087	114.5278	120.0769	234.6047
	0.17	1.7911	0.4094	114.4771	120.4263	234.9034

Parameter	Changed parameter values	$D(t) = \alpha\beta\, t^{\beta-1}$				
		T^*	L^*	C_r^*	C_s^*	TC^*
C_1	0.6	1.2395	0.0768	89.6886	164.1473	253.8359
	1	1.2329	0.0974	91.2974	163.9156	255.2131
	3	1.2231	0.1764	93.5950	163.8256	257.4206
	3.4	1.2225	0.1889	93.7301	163.8313	257.5615
C_2	2.4	1.2926	0.3392	88.9170	154.7639	243.6809
	4	1.2623	0.2414	90.8850	158.6169	249.5019
	12	1.2084	0.0994	93.7941	166.5581	260.3522
	13.6	1.2038	0.0889	93.9881	167.3230	261.3111
C_3	30	1.2105	0.1371	33.6055	165.7423	199.3478
	50	1.2153	0.1383	50.7040	165.1328	215.8368
	150	1.2346	0.1431	134.7147	162.7250	297.4397
	170	1.2379	0.1439	151.2816	162.3358	313.6175
A	60	1.1901	0.1321	90.2469	50.8095	141.0564
	100	1.2026	0.1352	90.8636	83.6220	174.4857
	300	1.2424	0.1451	95.5248	242.2830	337.8078
	340	1.2480	0.1465	96.6104	273.3199	369.9303
h_1	0.06	1.2274	0.1269	93.0385	163.3397	256.3782
	0.1	1.2267	0.1309	93.0078	163.5037	256.5115
	0.3	1.2248	0.1507	93.0054	164.0259	257.0313
	0.34	1.2245	0.1546	93.0248	164.0869	257.1116
h_2	0.03	1.2261	0.1371	92.9842	163.7170	256.7012
	0.05	1.2260	0.1382	92.9825	163.7474	256.7299
	0.15	1.2254	0.1436	92.9836	163.8799	256.8635
	0.17	1.2254	0.1446	92.9854	163.9029	256.8884

By analyzing the data in the Table 3 , we can conclude that with the values of parameters increasing, the values of $L^*, T^*, C_r^*, C_s^*, TC^*$ all increase or decrease by degrees. The market's demand rate is the comparison between the demand rate $D(t) = ae^{bt}$ and $D(t) = \alpha\beta\, t^{\beta-1}$ in Table 3.
The effect of various parameters on decision variable and total cost of inventory system is exhibited in the following tables and figures.
Here we observe in Table 4 that $T^*, L^*, C_r^*, C_s^*, TC^*$ are all moderately sensitive to changes in C_1 for the case $D(t) = ae^{bt}$ (See Fig.2).other side, for $D(t) = \alpha\beta\, t^{\beta-1}$, L* is highly sensitive and T^*, C_r^*, C_s^*, TC^* are less sensitive due to the changes in the value of C_1 (See Fig.3) .

Table 4. Effect of change in C_1 on the inventory

Change in C_1 (%)	$D(t) = ae^{bt}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	+17.3276	-23.3227	-22.0341	-9.5762	-15.6844
-50	+9.3276	-14.1067	-13.1041	-5.7937	-9.3780
+50	-4.7566	+8.9703	+8.1425	+3.5864	+5.8202
+70	-6.1212	+11.8211	+10.7187	+4.7049	+7.6535

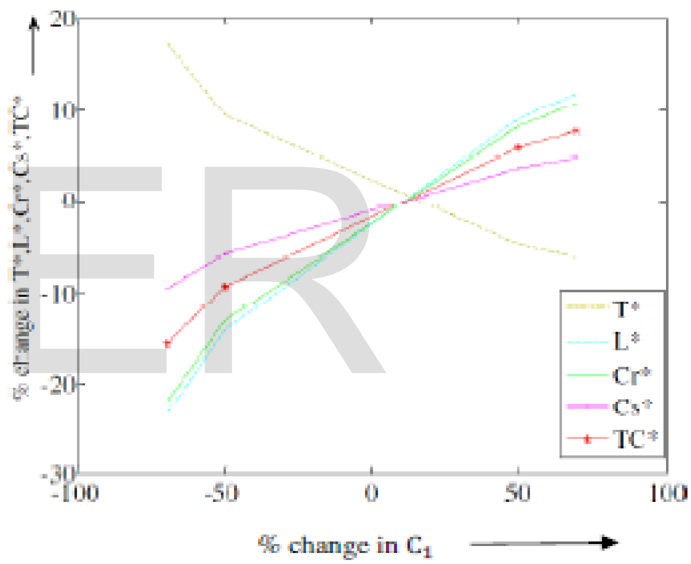

Fig. 2 The Effect of change in L^*, T^*, C_r^*, C_s^*, TC^* in the case of the demand rate $D(t) = ae^{bt}$ due to change in C_1 .

Change in C_1 (%)	$D(t) = \alpha\beta\, t^{\beta-1}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	+1.1259	-45.4933	-3.5411	0.2013	-1.1537
-50	+0.5874	-30.8730	-1.8109	0.0599	-0.6174
+50	-0.2121	+25.1952	+0.6601	0.0049	+0.2422
+70	-0.2611	+34.0667	+0.8054	0.0084	+0.2970

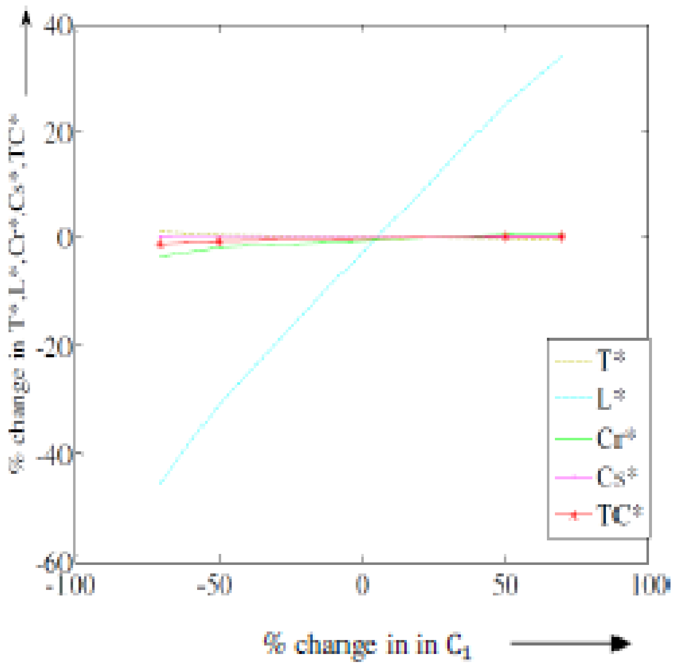

Fig. 3 The Effect of change in L^* , T^*, C_r^* , C_s^* , TC^* in the case of the demand rate $D(t)=\alpha\beta\, t^{\beta-1}$ due to change in C_1

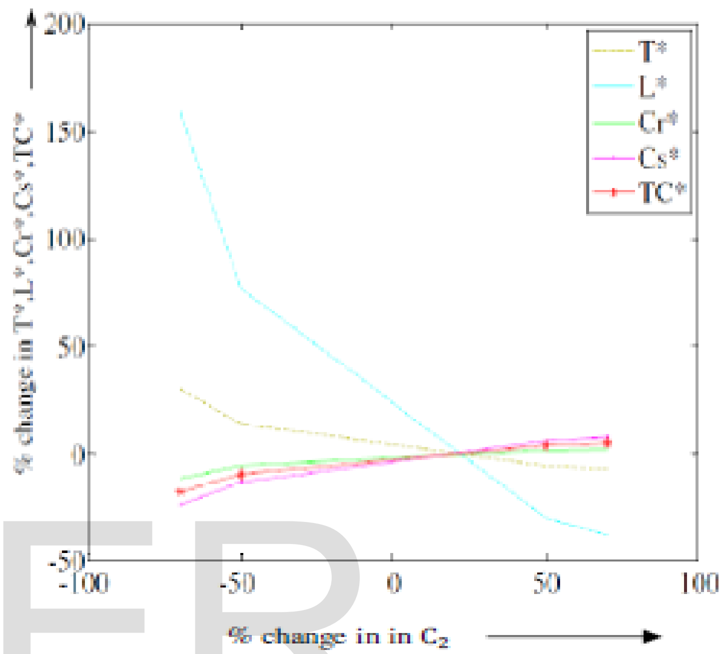

Fig. 4 The Effect of change in L^* , T^*, C_r^*, C_s^*, TC^* in the case of the demand rate $D(t) = ae^{bt}$ due to change in C_2 .

Here we notice in Table 5 that L^* is very highly sensitive and T^*, C_r^* , C_s^* , TC^* are slightly more sensitive for changes in the retailer's stockout cost C_2 in the case $D(t) = ae^{bt}$ (See Fig.4). On the other hand, L^* is very highly sensitive and T^*, C_r^* , C_s^* , TC^* are moderately sensitive to the changes in the retailer's stockout cost C_2 for $D(t) = \alpha\beta\, t^{\beta-1}$ (See Fig.5) .

Table 5. Effect of change in C_2 on the inventory

Change in C_2 (%)	$D(t) = ae^{bt}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	+29.7650	+159.0317	-11.3182	-23.8994	-17.7307
-50	+14.2308	+76.6527	-5.2670	-13.0244	-9.2208
+50	-5.4918	-29.9336	+1.7984	+6.1070	+3.9945
+70	-6.8620	-37.4048	+2.2113	+7.7380	+5.0282

Change in C_2 (%)	$D(t) = \alpha\beta\, t^{\beta-1}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	+5.4581	+140.7381	-4.3710	-5.5266	-5.1082
-50	+2.9860	+71.3272	-2.2544	-3.1746	-2.8414
+50	-1.4114	-29.4535	+0.8743	+1.6730	+1.3838
+70	-1.7867	-36.9056	+1.0829	+2.1399	+1.7572

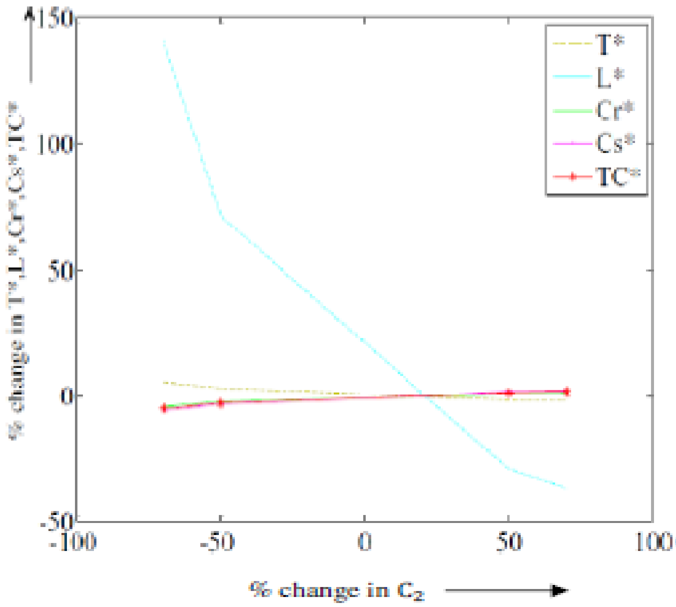

Fig.5 The Effect of change in L^*, T^*, C_r^*, C_s^*, TC^* in the case of the demand rate $D(t)=\alpha\beta t^{\beta-1}$ due to change in C_2.

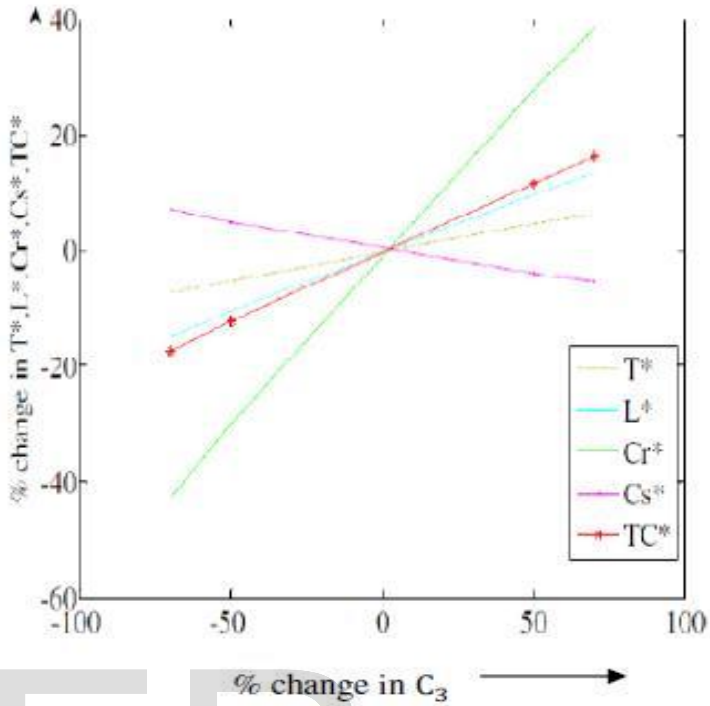

Fig. 6 The Effect of change in L^*, T^*, C_r^*, C_s^*, TC^* in the case of the demand rate $D(t) = ae^{bt}$ due to change in C_3.

Here we observe in Table 6 that C_r^* is comparatively sensitive but T^*, L^*, C_s^*, TC^* are all moderately sensitive to changes in the retailer's setup cost C_3 for the case $D(t) = ae^{bt}$ (See Fig.6). other side, for $D(t) = \alpha\beta\, t^{\beta-1}$, C_r^* is comparatively sensitive, L* & TC^* are moderately sensitive but T^* & C_s^* are less sensitive due to the changes in the retailer's setup cost C_3 (See Fig.7).

Table 6. Effect of change in C_3 on the inventory

Change in C_3 (%)	$D(t) = ae^{bt}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	-7.4245	-14.8685	-42.7016	+7.1272	-17.3171
-50	-5.1688	-10.4448	-29.9538	+4.8369	-12.2210
+50	+4.6174	+9.7075	+27.8019	-3.9120	+11.6376
+70	+6.3440	+13.4185	+38.4378	-5.2835	+16.1534

Change in C_3 (%)	$D(t) = \alpha\beta t^{\beta-1}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	-1.2401	-2.6969	-63.8577	+1.1750	-22.3720
-50	-0.8485	-1.8453	-45.4685	+0.8029	-15.9510
+50	+0.7261	+1.5614	+44.8838	-0.6669	+15.8260
+70	+0.9953	+2.1292	+62.7013	-0.9045	+22.1258

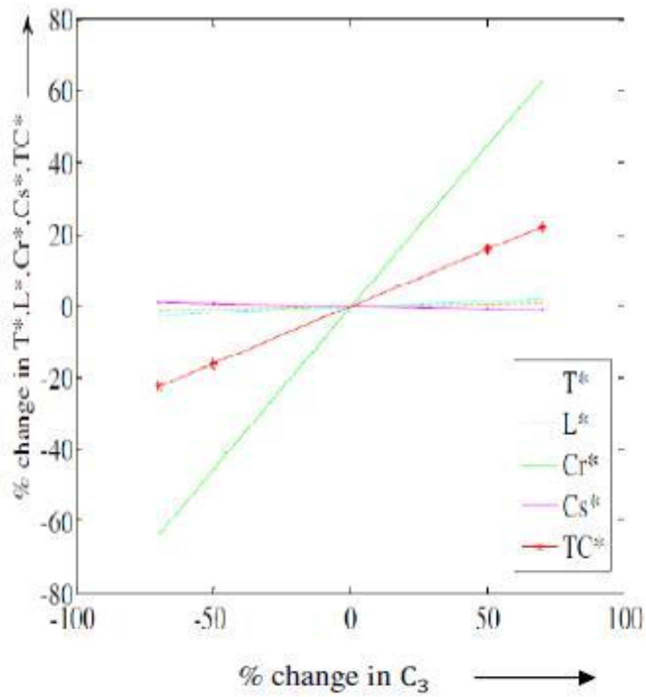

Fig. 7 The Effect of change in L^*, T^*, C_r^*, C_s^*, TC^* in the case of the demand rate $D(t)=\alpha\beta t^{\beta-1}$ due to change in C_3

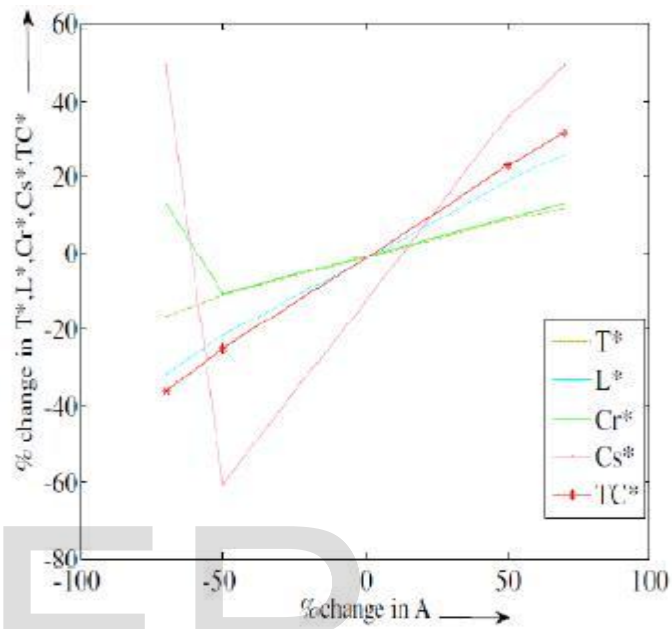

Fig. 8 The Effect of change in L^*, T^*, C_r^*, C_s^*, TC^* in the case of the demand rate $D(t) = ae^{bt}$ due to change in A .

Here we notice in Table 7 that L^*, C_s^*, TC^* are highly sensitive and T^*, C_r^* are comparatively less sensitive for changes in the supplier's setup cost A in the case $D(t) = ae^{bt}$ (See Fig.8). On the other hand, C_s^*, TC^* are highly sensitive and T^*, L^*, C_r^* are slightly sensitive to the changes in the supplier's setup cost A for $D(t) = \alpha\beta\, t^{\beta-1}$ (See Fig.9) .

Table 7. Effect of change in A on the inventory

Change in A (%)	$D(t) = ae^{bt}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	-16.5646	-31.8014	+13.1453	+49.0091	-36.2261
-50	-11.0727	-21.8235	-10.7728	-60.7114	-25.1764
+50	+8.8225	18.8253	+9.3096	+35.7668	+22.7946
+70	+11.9416	25.7557	+13.1453	+49.0091	+31.4248

Change in A (%)	$D(t) = \alpha\beta t^{\beta-1}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	-2.9045	-6.2456	-2.9407	-68.9841	-45.0712
-50	-1.8846	-4.0454	-2.2774	-48.9542	-32.0535
+50	+1.3625	+2.9808	+2.7356	+47.8981	+31.5458
+70	+1.8194	+3.9744	+3.9032	+66.8441	+44.0546

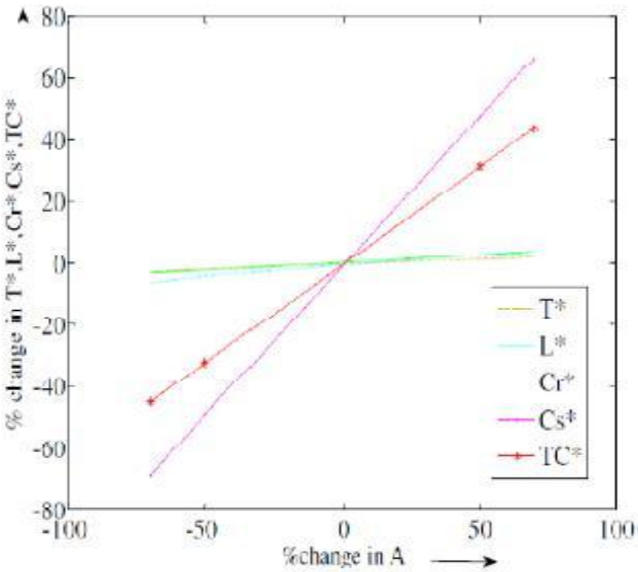

Fig. 9 The Effect of change in L^* ,T^*, C_r^* , C_s^* , TC^* in the case of the demand rate $D(t)= \alpha\beta t^{\beta-1}$ due to change in A.

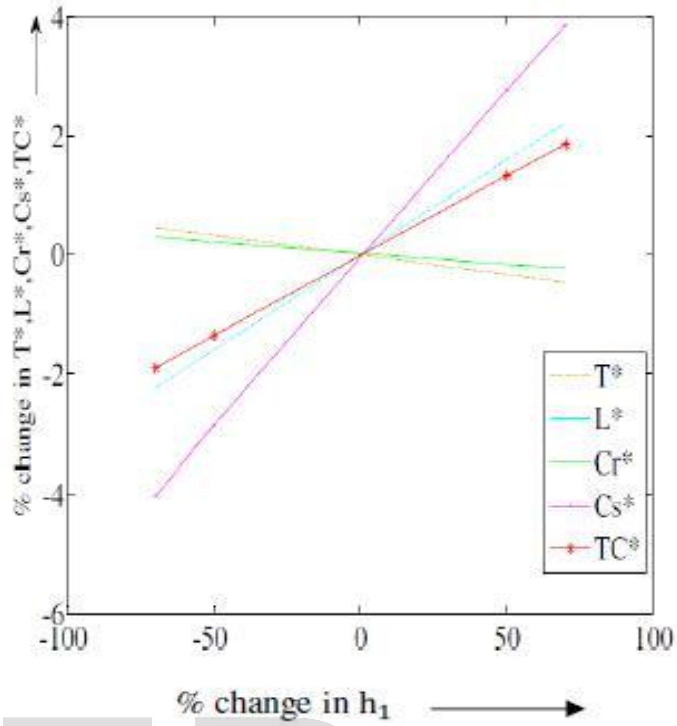

Fig. 10 The Effect of change in L^* ,T^*, C_r^*, C_s^* , TC^* in the case of the demand rate $D(t) = ae^{bt}$ due to change in h_1 .

Here we observe in Table 8 that L^*, C_s^* , TC^* are slightly sensitive but T^*, C_r^* are not much sensitive to changes in the value of h_1 for the case $D(t) = ae^{bt}$ (See Fig.10). other side, for $D(t) = \alpha\beta\ t^{\beta-1}$, L^* is moderately sensitive but T^*, C_r^* ,C_s^* ,TC^* are not generally much sensitive due to the changes in the value of h_1 (See Fig.11) .

Table 8. Effect of change in h_1 on the inventory

Change in h_1 (%)	$D(t) = ae^{bt}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	+0.4512	-2.2364	+0.3039	-4.0273	-1.9036
-50	+0.3230	-1.5974	+0.2084	-2.8591	-1.3550
+50	-0.3286	1.5974	-0.1659	+2.7725	+1.3317
+70	-0.4623	2.2118	-0.2207	+3.8576	+1.8580

Change in h_1 (%)	$D(t) = \alpha\beta t^{\beta-1}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	+0.1387	-9.9361	+0.0616	-0.2917	-0.1637
-50	+0.0816	-7.0972	+0.0286	-0.1916	-0.1118
+50	-0.0734	+6.9553	+0.0260	+0.1272	+0.0906
+70	-0.0979	+9.7232	+0.0469	+0.1645	+0.1218

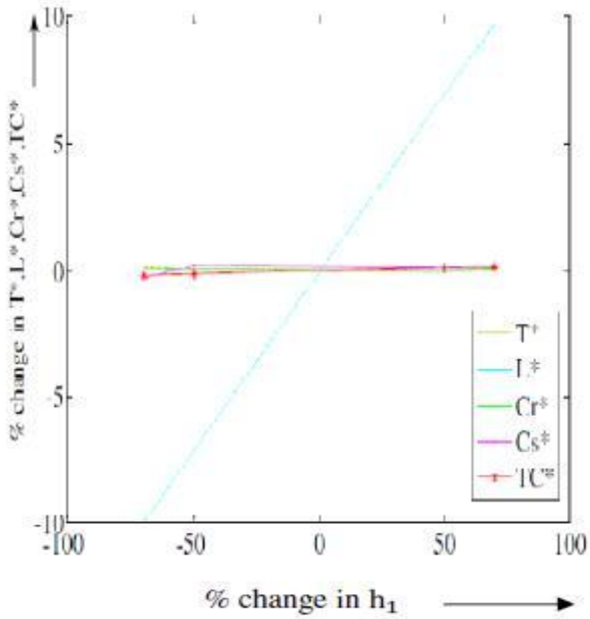

Fig. 11 The Effect of change in L^*, T^*, C_r^*, C_s^*, TC^* in the case of the demand rate $D(t)=\alpha\beta t^{\beta-1}$ due to change in h_1.

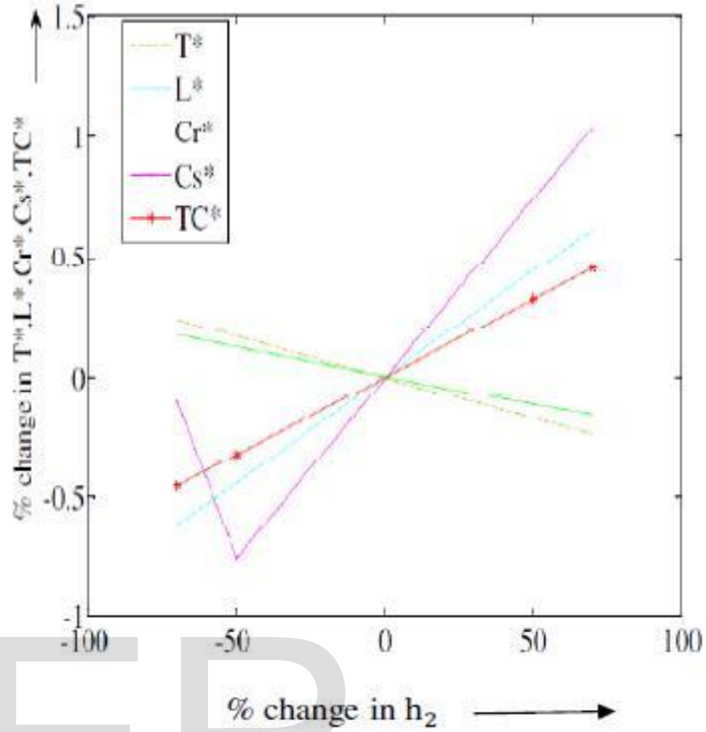

Fig. 12 The Effect of change in L^*, T^*, C_r^*, C_s^*, TC^* in the case of the demand rate $D(t) = ae^{bt}$ due to change in h_2.

Here we notice in Table 9 that $T^*, L^*, C_r^*, C_s^*, TC^*$ are very slightly sensitive for changes in the value of h_2 in the case $D(t) = ae^{bt}$ (See Fig.12). On the other hand, but L* is slightly sensitive but T*, C_r^*, C_s^*, TC^* are not generally much sensitive due to the changes in the value of h_2 for $D(t) = \alpha\beta\ t^{\beta-1}$ (See Fig.13).

Table 9. Effect of change in h_2 on the inventory

Change in h_2 (%)	$D(t) = ae^{bt}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	+0.2395	-0.6144	+0.1734	-1.0606	-0.4556
-50	+0.1671	-0.4424	+0.1224	-0.7549	-0.3247
+50	-0.1727	+0.4424	-0.1153	+0.7418	+0.3216
+70	-0.2395	+0.6144	-0.1595	+1.0350	+0.4493

Change in h_2 (%)	$D(t) = \alpha\beta t^{\beta-1}$ % change in				
	T^*	L^*	C_r^*	C_s^*	TC^*
-70	+0.0326	-2.6969	+0.0032	-0.0613	-0.0380
-50	+0.0245	-1.9163	+0.0014	-0.0428	-0.0268
+50	-0.0245	+1.9163	+0.0026	+0.0381	+0.0252
+70	-0.0245	+2.6260	+0.0045	+0.0521	+0.0349

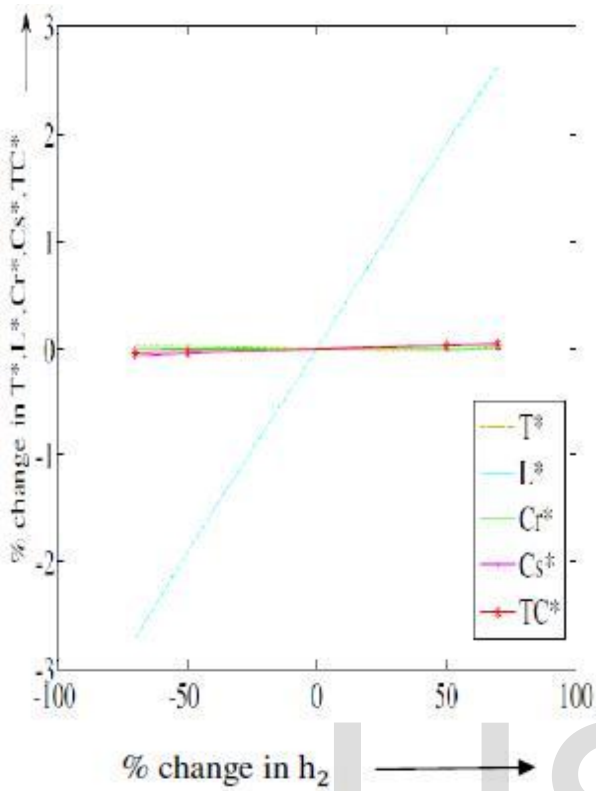

Fig.13 The Effect of change in $L^*, T^*, C_r^*, C_s^*, TC^*$ in the case of the demand rate $D(t)=\alpha\beta t^{\beta-1}$ due to change in h_2.

8 Conclusion

This paper studies the optimal order strategy of the supplier and the retailer. When the supplier holds a monopolistic status, in which he controls not only products' prices but also the lead time, he actually controls the retailer's order cycle time. Such a control only benefits the supplier, not the retailer and the whole supply chain. If we take h_2=0 in Supplier's holding cost and n=0 in retailer's holding cost, then both holding cost become constant. In that case supplier will gain more profit compare to retailer. Taking holding cost and demand rate constant, some Researchers have developed a model where it is observed that supplier's average total cost decreases and the retailer's average total cost increases. Such a buyer-Vendor relationship greatly goes against the retailer and hardly does any good to the distribution channel system. As a result, if the supplier makes some advisable concessions, then both sides can win. In this paper, we take time-dependent holding cost for both supplier and retailer. Considering Exponential and Weibull demand rate, we notice that retailer's profit is higher than supplier's profit expect some of the results. So, retailer becomes more beneficial than the supplier in this type of model. In future the obtained optimal solutions can be improved by using different algebraic procedure or Geometric Programming Problem approach for the case of Posynomial functions which arises in Engineering problems.

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